# A steady-state field-circuit model based on the magnetostatic analysis for PM-BLDC motors driven from $120^o$ and $180^o$ quasi-square wave inverters

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**A consistent and computationally efficient finite element model for simulation of the steady-state operation of permanent magnet brushless DC motors driven from quasi-square wave inverters is proposed. The algorithm combines the magnetostatic finite element and the steady-state time-periodic circuit models with time averaging. A weak field-circuit coupling is established through the effective constant current and lumped parameters extracted at various loading conditions. The performance characteristics, determined via the proposed model for the two different motors, are comparable to those obtained from the comprehensive time-stepping finite element model, with the execution time being approximately hundred times shorter for the former.**

***Index Terms*— Brushless motors, finite element analysis, steady-state analysis.**

## I. INTRODUCTION

UNLIKE synchronous motors driven from sinusoidal sources, for which there are various models proposed to date [1, 2], brushless DC motors (BLDC) driven from quasi-square wave voltage source inverters do not have the complementary steady-state models based on the magnetostatic analysis. This is not surprising as those are the nonsinusoidal-flux machines with apparent ripple of the electromagnetic torque, whilst the patterns of their voltage and current waveforms considerably change with loading [3].

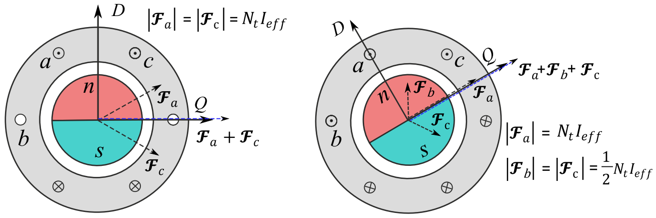
It should be noticed that the state-of-the-art algorithms for the steady-state simulation of AC machines based on either the harmonic ballance finite element method [4], or the time-periodic steady-state FEM [5], are computationally too demanding to be applicable in the design routines. For such the purpose it would be good to have a faster, possibly the same reliable, but simpler technique.

The model proposed here matches the most of these requirements. It allows calculation of performance characteristics represented by the time-averaged quantities. The complex patterns of currents waveforms are represented by the constant effective current determined from auxiliary circuit model. Nonlinearity, armature reaction and stack skew are taken into account. The model is confirmed to be suitable for the three-phase surface-mounted permanent-magnet motors driven from either $120^{O}$ or $180^{O}$ quasi-square wave voltage source inverters.

## II. STEADY-STATE MODEL OF BLDC MOTOR

### A. Magnetostatic model of machine

This model is used for calculation of electromagnetic torque and extration of lumped parameters. The phase windings are supplied with the effective current $I_{eff}$ which is to be calculated further from the auxilary circuit model. Depending on inverter type, the phase windings are supplied as depicted in Fig. 1a or 1b, whilst the rotor is positioned so as to align the axis $Q$ with the phasor of winding total mmf.



a) b)

Fig. 1. Orientation of main flux due to magnets and the way of supplying phase windings in magnetostatic analysis: a) $120^o$ inverter - phase $b$ is not supplied, b) $180^o$ inverter - all phases are supplied; $\mathcal{F}_a, \mathcal{F}_b, \mathcal{F}_c$ are phasors of magnetomotive forces, and $N_t$ is the number of turns per phase.

### B. Steady-state time-periodic circuit model of inverter

This auxiliary circuit model is used for calculation of $I_{eff}$. The symmetric two-level inverter is considered. The model consists of ordinary differential equations that represent behavior of switching elements. The equations for the motor windings use lumped parameters determined form the field model in Fig. 1. It is assumed that elements of the inductance matrix vary only with $I_{eff}$ and the phase back-emf is a sinusoid of magnitude $E_{eff} = \lambda_{pm} \omega_e E_{pk}/E_1$, with $\lambda_{pm}$ being the phase linkage flux due to magnets determined under load, $\omega_e$ the electrical angular speed, and $E_{pk}$ and $E_1$, respectively the peak value and the magnitude of the fundamental harmonic of phase back-emf calculated at $I_{eff} = 0$. This is a circumvention of the problem involving $\lambda_{pm}$, determined from a single magnetostatic solution, which does not contain information about peak value of the back-emf waveform. For example, in motors with quasi-trapezoidal back-emf waveform the peak value is lower than magnitude of the fundamental harmonic, and the ratio $E_{pk}/E_1$ is a measure of this relationship. The resultant system of equations is solved imposing conditions of time periodicity so as to obtain only the steady-state waveforms of currents. For both ($120^{O}$ and $180^{O}$) inverters the effective current $I_{eff}$ is calculated via averaging of current that flows through the DC-link wire.

### *C. Algorithm of computations*

The computations must be carried out iteratively as $I_{eff}$ is never known in advance and the extracted lumped parameters are dependent on this current. Prior to initiation of the iterations a single half-period of the phase back-emf must be calculated so as to determine $E_{pk}/E_1$. This calculation is, however done only once for each motor at $I_{eff}$ = 0. A flowchart diagram of the algorithm is depicted in Fig. 2.

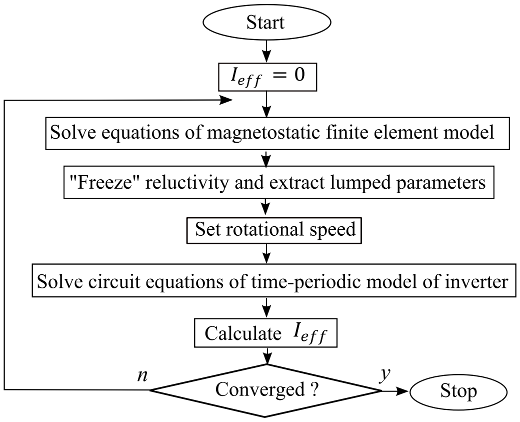


Fig. 2 Algorithm based on weak coupling between magnetostatic model of machine and time-periodic circuit model of voltage source inverter.

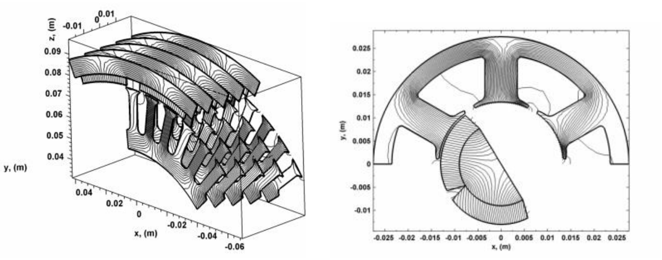
It requires between 4 and 6 iterations to converge. The measure of the convergence is the relative change in electromagnetic torque calculated via the Maxwell stress tensor.

## III. COMPUTATIONS

The model was tested on the two motors with specifications given in Tab. 1 operating with 120° and 180° inverters. The same characteristics were determined point-by-point, using the time-stepping finite element models. The experimental data is also presented for the 120° inverters. Results are shown in Tab. 2 and Figs. 3 - 4.

TABLE I
BASIC SPECIFICATIONS OF CONSIDERED PM-BLDC MOTORS

| Parameter | Motor 1 | Motor 2 |
|---|---|---|
| Rating | 180 W, 170 rpm | 180 W, 4000 rpm |
| Winding type | 14-pole, distributed, one-layer | 4-pole, concentrated, one-layer |
| Stack skew | one slot-pitch | None |
| Magnets | High-energy NdFeB | Low-energy ceramic |



a) b)

Fig. 3 Magnetic flux plots at rated speed for 120° inverter: a) motor 1 (multi mesh-slice approximation of stack skew is used), b) motor 2. For both models the *Q* axis is positioned as shown in Fig. 1.

TABLE II
COMPARISON OF EXECUTION TIMES FOR DIFFERENT MODELS REGARDING DETERMINATION OF TWELVE POINTS OF CHARACTERISTICS IN FIG. 4.

| | Motor 1 | |
|---|---|---|
| Model used: | 120° inverter | 180° inverter |
| Time-stepping | 15 h, 33 min | 14 h, 43 min. |
| Proposed | ≈11 min | ≈ 8 min. |
| | Motor 2 | |
| Model used: | 120° inverter | 180° inverter |
| Time-stepping | 6 h, 34 min | 5 h, 56 min. |
| Proposed | ≈ 5 min | ≈ 4 min. |

As one can notice, the results diverge within 12 % in the entire range of speed, whereas the benefits regarding a very short execution time of the proposed model are very clear.

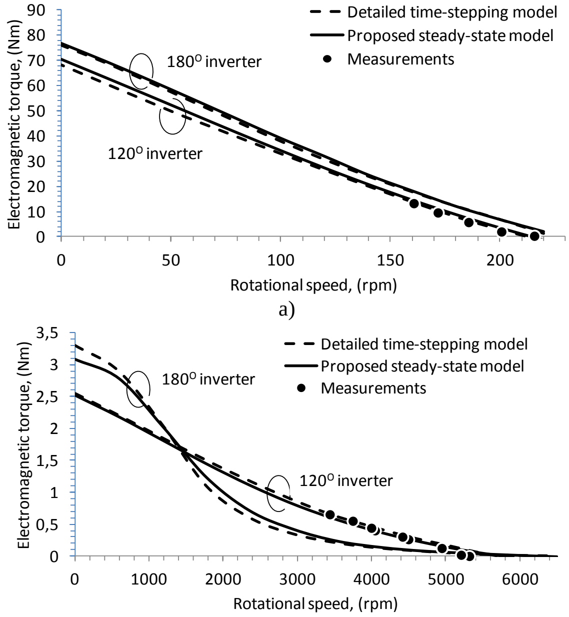


Fig. 4. Comparison of output characteristics obtained at rated voltage from different models and measured: a) for motor 1, b) for motor 2.

## IV. CONCLUSION

The correct results of computations have shown that the model is applicable to motors having various back-emf profiles. This is because the effective current $I_{eff}$ depends more on the magnitude of back-emf than on its harmonic spectrum. This will be outlined more comprehensively in the full paper.

## V. REFERENCES

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